Consider the problem of finding the sum of all integers from 1 to nnn that are divisible by 3 or 5. Specifically, let's solve this for n=1000n = 1000n=1000. This problem, often referred to as the "Multiples of 3 and 5" problem, is interesting due to its application in programming and algorithmic thinking, particularly in optimizing loops and understanding arithmetic sequences.

**Why the Problem is Interesting:** This problem is fascinating because it combines basic arithmetic operations with the concept of inclusion-exclusion in set theory. It also has practical implications in computational problem-solving, as it requires an efficient approach to handle large numbers. The challenge lies in correctly identifying and summing the multiples, ensuring that common multiples of 3 and 5 are not double-counted.

**Solution Process:**

1. **Identifying Multiples:** To find the multiples of 3, we consider the series 3, 6, 9, ..., up to the largest multiple less than or equal to 1000. The number of terms in this sequence is given by ⌊10003⌋\left\lfloor \frac{1000}{3} \right\rfloor⌊31000​⌋, which results in 333 terms. Similarly, the number of multiples of 5 is ⌊10005⌋=200\left\lfloor \frac{1000}{5} \right\rfloor = 200⌊51000​⌋=200.
2. **Summing the Multiples:** The sum of the first mmm multiples of a number kkk can be found using the formula:  
   Sum=k×m(m+1)2\text{Sum} = k \times \frac{m(m + 1)}{2}Sum=k×2m(m+1)​  
   Therefore, the sum of multiples of 3 up to 1000 is:  
   3×333×3342=3×55611=1668333 \times \frac{333 \times 334}{2} = 3 \times 55611 = 1668333×2333×334​=3×55611=166833  
   Similarly, the sum of multiples of 5 up to 1000 is:  
   5×200×2012=5×20100=1005005 \times \frac{200 \times 201}{2} = 5 \times 20100 = 1005005×2200×201​=5×20100=100500
3. **Handling Overlap (Multiples of 15):** The multiples of both 3 and 5 (i.e., multiples of 15) are counted twice in the above sums. We must subtract these to avoid double-counting:  
   Multiples of 15=15×66×672=15×2211=33165\text{Multiples of 15} = 15 \times \frac{66 \times 67}{2} = 15 \times 2211 = 33165Multiples of 15=15×266×67​=15×2211=33165
4. **Final Calculation:** The total sum of integers from 1 to 1000 that are divisible by 3 or 5 is:  
   166833+100500−33165=234168166833 + 100500 - 33165 = 234168166833+100500−33165=234168

**Critique and Improvement:** The solution provided is correct and efficient, leveraging the arithmetic series formula to avoid looping through all numbers up to 1000. However, it can be further optimized by recognizing that the formula for the sum of multiples can be simplified using the formula for the sum of an arithmetic series:

S=n2(a+l)S = \frac{n}{2} (a + l)S=2n​(a+l)

Where aaa is the first term, lll is the last term, and nnn is the number of terms. For instance, for multiples of 3, a=3a = 3a=3, l=999l = 999l=999 (the largest multiple of 3 less than 1000), and n=333n = 333n=333. This formula provides a direct way to compute the sum without separately counting terms and then applying another formula.

Additionally, the use of Python or another programming language could automate this process, especially for larger values of nnn. A simple loop or a list comprehension can sum these multiples quickly, which is particularly useful for dynamic ranges.

**Conclusion:** The "Multiples of 3 and 5" problem is a classic example that demonstrates efficient mathematical techniques and the importance of considering overlapping sets. The initial solution, using the sum of arithmetic series formula, is efficient and avoids unnecessary computations. However, the solution can be further refined and automated, highlighting the interplay between mathematical understanding and practical application in programming.

References:

Stroud, K. A., & Booth, D. J. (2013). *Engineering Mathematics* (7th ed.). Palgrave Macmillan.

Knuth, D. E. (1997). *The Art of Computer Programming, Volume 1: Fundamental Algorithms* (3rd ed.). Addison-Wesley.